

FLA (Fall 2026) – Assignment 2

Name: _____ Dept: _____

Grade: _____ ID: _____

Due: Sep. 30, 2026

Problem 1

Prove that the following languages are not regular using **the pumping lemma**.

- a. $\{\omega \in \{0, 1\}^* \mid \omega = \omega^R\}$
- b. $\{uv \mid u, v \in \{0, 1\}^*, |u|_0 = |v|_0 \wedge |u|_1 = |v|_1\}$
- c. $\{0^{n_1}10^{n_2}1 \cdots 0^{n_k}1 \mid k \geq 1, n_1, \dots, n_k \geq 1, n_i \neq n_j \text{ for all } 1 \leq i < j \leq k\}$
- d. $\{0^{2^m+2^n} \mid m, n \geq 0\}$

Solution.

Problem 2

Prove or disprove the following statements (All languages mentioned below are over alphabet Σ):

- a. If $A \cup B$ and $A \cap B$ are regular languages, then both A and B are regular languages.
- b. If A is a nonempty regular language and B is not a regular language, then AB is not regular.
- c. For any set of regular languages S , $\bigcup S$ (the union of all the elements in S) is a regular language.
- d. An infinite regular language L can be split into two infinite disjoint, nonempty, regular subsets L_1 , L_2 .

Solution.

Problem 3

Let R be a regular language over the alphabet $\{0, 1\}$. For each string in R , delete exactly one 1 from each maximal block of consecutive 1's of positive even length, leaving all other symbols unchanged. For example, the string 11011101111 becomes 101110111. Prove that the language consisting of all resulting strings is regular, **using the closure properties of regular languages**.

Solution.

Problem 4 (Bonus.)

Prove that the class of regular languages is closed under the following operation:

$$\text{cycle}(L) = \{w \mid w = xy \text{ for some } x, y \in \Sigma^* \text{ such that } yx \in L\}.$$

For example, if $L = \{01, 011\}$, then $\text{cycle}(L) = \{01, 10, 011, 110, 101\}$. (**Hint:** Starting from a DFA for L , construct an ϵ -NFA for $\text{cycle}(L)$.)

Solution.