

FLA (Fall 2026) – Assignment 4

Name: _____ Dept: _____

Grade: _____ ID: _____

Due: Oct. 28, 2026

Problem 1

Given the following context-free grammar G :

$$S \rightarrow AB \mid BC$$

$$A \rightarrow BA \mid a$$

$$B \rightarrow AC \mid b$$

$$C \rightarrow BB \mid a$$

Use the **CYK algorithm** to determine whether the string *baabab* belongs to $L(G)$. Show the complete CYK table.

LLM Usage (select one):

☐ None ☐ Assistance only ☐ Mostly/entirely generated

If used, briefly describe how: _____

Solution.

Problem 2

Prove that each of the following languages is not context-free using the **pumping lemma for context-free languages**.

- a. $L = \{a^m b^m c^n \mid 1 \leq m < n\}$.
- b. $L = \{a^{mn} b^m c^n \mid m, n \geq 1\}$.
- c. $L = \{w \in \{a, b\}^* \mid w = w^R, |w|_a = |w|_b\}$.
- d. $L = \{x\#y \mid x, y \in \{a, b\}^*, x \text{ is a substring of } y\}$.

LLM Usage (select one):

☐ None ☐ Assistance only ☐ Mostly/entirely generated

If used, briefly describe how: _____

Solution.

Problem 3

For any context-free language L over an alphabet Σ , determine whether each of the following statements concerning the **closure properties** of context-free languages is **True** or **False**. If your answer is **True**, give an explanation. If your answer is **False**, give a counterexample and **explain why it disproves the statement**.

- a. $E(L) = \{w \in L \mid |w|_a \text{ is even}\}$ is context-free. (here $\Sigma = \{a, b\}$)
- b. $\text{init}(L) = \{w \in \Sigma^* \mid wx \in L \text{ for some } x \in \Sigma^+\}$ is context-free.
- c. $\text{root}(L) = \{w \in \Sigma^* \mid ww \in L\}$ is context-free.
- d. $\text{min}(L) = \{w \in L \mid \text{no proper prefix of } w \text{ belongs to } L\}$ is context-free.

LLM Usage (select one):

☐ None ☐ Assistance only ☐ Mostly/entirely generated

If used, briefly describe how: _____

Solution.

Problem 4

For the following statements concerning the **decision properties** of context-free languages, determine whether each is **True** or **False**. If your answer is **True**, provide a proof. If your answer is **False**, give a counterexample and **explain why it disproves the statement**.

- a. If a context-free grammar contains a recursive production (i.e., a production of the form $A \rightarrow \alpha A \beta$, where $\alpha, \beta \in (V \cup \Sigma)^*$), then the language generated by the grammar must be infinite.
- b. For any context-free language L and any pumping length p for L , the language L is infinite if and only if there exists a string $w \in L$ such that

$$p \leq |w| < 2p.$$

- c. Every infinite context-free language contains an infinite regular subset.

LLM Usage (select one):

☐ None ☐ Assistance only ☐ Mostly/entirely generated

If used, briefly describe how: _____

Solution.